

II B. Tech I Semester Regular Examinations, Feb/March - 2022
RANDOM VARIABLES AND STOCHASTIC PROCESSES
 (Com to ECE, ECT)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

- 1 a) State and prove the properties of cumulative distribution function (CDF) of X. [7M]
 b) If the communicative distribution function of a random variable X is given by [7M]

$$f_X(x) = \begin{cases} \frac{x^2}{3} & -1 \leq x, y \leq 2 \\ 0 & \text{Else where} \end{cases}$$

Find $P(0 < X < 1)$ and $F_X(x)$?

Or

- 2 a) Define conditional probability distribution function and write the properties. [7M]
 b) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball? [7M]

- 3 a) State and prove the Chebychev's inequality theorem. [7M]
 b) A Gaussian random variable with variance 10 and mean 5 is transformed to $y = e^x$. Find the pdf of y. [7M]

Or

- 4 a) Show that any characteristic function $\Phi_X(\omega)$ satisfies $\Phi_X(\omega) \leq \Phi_X(0) = 1$. [7M]
 b) A random variable X is defined by density function [7M]

$$f_X(x) = \begin{cases} 3x^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find $E[X]$, $E[3X]$ and $E[X^2]$.

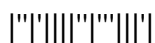
- 5 a) Explain central limit theorem with equal and unequal distributions. [7M]
 b) If X and Y are independent, show that $E[XY] = E[X] E[Y]$. [7M]

Or

- 6 a) Two statistically independent random variables X and Y have respective densities $f_X(x) = 5e^{-5x}u(x)$, $f_Y(y) = 2e^{-2y}u(y)$. Find the density of the sum $W = X + Y$. [7M]
 b) Gaussian random variables X and Y have first and second order moments $m_{10} = -1.1$, $m_{20} = 1.16$, $m_{01} = 1.5$, $m_{02} = 2.89$, $R_{XY} = -1.724$. Find C_{XY} , ρ . [7M]

- 7 a) Derive the relation between correlation and covariance of two random variables X and Y. [7M]
 b) A random process $X(t) = A \cos(\omega_c t + \theta)$ where θ is a random variable uniformly distributed in the range $(0, 2\pi)$. Show that the process is ergodic in mean and correlation sense. [7M]

Or



- 8 a) The auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 625 + \frac{16}{1+36\tau^2}$. Find mean and variance of the random process. [7M]
- b) Explain about Poisson random processes. [7M]
- 9 a) Derive the relationship between cross-power spectral density and cross correlation function. [7M]
- b) Define the following systems. [7M]
- (i) Band pass process
 - (ii) Band – Limited process
 - (iii) Narrow band process
 - (iv) Band – Limited Band pass process
- Or
- 10 a) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$. [7M]
- b) Find the mean and mean- square values of output $y(t)$ of an LTI system with input $x(t)$. Assume that $x(t)$ is a WSS process. [7M]

